Sample Exploration Problem

Our application includes three multi-part exploration problems, similar to the one below. Applicants will have 2 - 3 hours to work on the problems and write up their solutions. We suggest that applicants choose just one or two problems to focus on during this time, and we're interested in seeing their progress even if they aren't able to solve all of the parts.

Trapezoidal Numbers[1]

If I asked you to compute the sum $3+4+5$, one way that you could do it would be drawing a trapezoid and counting the number of dots (on the left) or the number of boxes (on the right):

Because of this, the answer (12) is called a trapezoidal number. Other trapezoidal numbers include $4+5+6+7+8=30$ and $6+7=13$. A trapezoidal number has to have at least two rows when represented as a trapezoid, and must have the pattern where each row has one more dot or box than the previous row.

(a) What numbers can be written as 2-row trapezoidal numbers, like $6+7=13$?

(b) What about 3-row trapezoidal numbers? 4-row trapezoidal numbers?

(c) Can you give a general rule, so that we can easily tell whether 192 is a 12-row trapezoidal number? Explain your rule and why it works, and use it to find the answer for 192.

(d) Can you name a number above 100 that is not trapezoidal, no matter what number of rows you try? How do you know it can't be trapezoidal?

(e) Based on the large number you found, write down a statement of the form: "If n is _____, then n is not a trapezoidal number."

(f) Can you show that the statement you wrote down in part (e) is true?

(g) Is there anything else you notice or wonder about trapezoidal numbers? What other questions can you ask? Can you answer them?

[1]Based off of a Julia Robinson Math Festival (jrmf.org) problem.